Isotope notations: %s, δ values, and ε values

Geochemists have used various means to express the ratios of isotopes found in various materials. Sometimes they have just reported the raw ratios of two isotopes, as is commonly done today for ^{87}Sr and ^{86}Sr. More commonly, they have used notations that express the difference of the observed ratio in a sample from the ratio found in some standard material. This page tries to explain why different such notations are used for different pairs of isotopes.

Let's begin with a simple case, that of hydrogen. Geochemists compare the ratio of ^{2}H (deuterium) and ^{1}H in a sample to the ratio of those two isotopes in a standard (which happens to be Standard Mean Ocean Water, or SMOW). They express that comparison as a per cent difference. Thus the formulation is

$$\frac{{}^{2}H/{}^{1}H_{sample} - {}^{2}H/{}^{1}H_{standard}}{{}^{2}H/{}^{1}H_{standard}} \times 100 =$$

$$= \frac{R_{sample} - R_{standard}}{R_{standard}} \times 100 = \%\ \text{difference}$$

much as one would calculate any per cent difference.

δ (delta) values use exactly the same logic, except they usually express difference in parts per thousand (per mil, or ‰) rather than in parts per hundred (per cent, or %)). Thus the formulation is

$$\delta = \frac{R_{sample} - R_{standard}}{R_{standard}} \times 1000 = ‰\ \text{difference}$$

ε (epsilon) values likewise use exactly the same logic, except they express difference in parts per ten thousand Thus the formulation is

$$\varepsilon = \frac{R_{sample} - R_{standard}}{R_{standard}} \times 10{,}000$$

So why do we use %s in some cases, δs in others, and εs in yet others? The answer has mostly to do with relative difference in the masses of the two isotopes. For hydrogen, the relative difference in mass of the two nuclides (2 and 1) is 100%, so there are large fractionations that are conveniently expressed as %s. For elements where the difference in mass is less pronounced (e.g., for oxygen it's 12.5%), fractionations are smaller, and so %s would all be numbers of the order 0.XX. It's more convenient to report those as δ values in ‰, where they become X.X. For elements like hafnium, the difference in masses is very small, so there's little chemical fractionation, and instead the isotope ratios are of interest because of radioactive decay that produces one of them. As a result, %s would be of the order of 0.0XX. Thus it becomes convenient to express the results in parts per ten thousand, again as X.X.

That's what the plot below shows: the notations that we use are largely a function of the relative difference in masses of the nuclides reported. Thus you'll always see "$\delta^{18}O$" but "$\varepsilon^{143}Nd$".