The delta symbol in stable isotope geochemistry

Stable isotope geochemistry involves measuring, and therefore expressing, ratios between two isotopes of a given element. Two common examples are the isotopes of oxygen and carbon, ^{18}O & ^{16}O and ^{13}C & ^{12}C, where in each pair the first is the heavier and more rare isotope and the second is the lighter and far more abundant. We express the ratio of the two isotopes in a sample relative to some standard material that is accepted world-wide as the benchmark against which the ratio in samples will be expressed.

The delta symbol is a way to express the relative difference of isotope ratios between sample and standard. The examples at right use ratios of oxygen isotopes and thus $\delta^{18}O$, but we could as easily use carbon isotopes and $\delta^{13}C$, or other isotopic pairs as well. The two equations at right are just algebraic transformations of each other, but examining both is useful in understanding delta values.

$$\delta^{18}O_{sample} = \frac{\left(\frac{^{18}O}{^{16}O}\right)_{sample} - \left(\frac{^{18}O}{^{16}O}\right)_{standard}}{\left(\frac{^{18}O}{^{16}O}\right)_{standard}} \times 1000$$

This formulation is instructive in showing the basic idea of a relative difference in ratios: subtracting the ratio in the standard from that in the sample, and dividing by the ratio in the standard, expresses how much the sample differs from the standard. Because these differences for C and O are very small, we don't multiply by 100 to give a percent (%) difference. Instead, we multiply by 1000 to express the difference in per mil (‰).

$$\delta^{18}O_{sample} = \left(\frac{\left(\frac{^{18}O}{^{16}O}\right)_{sample}}{\left(\frac{^{18}O}{^{16}O}\right)_{standard}} - 1 \right) \times 1000$$

This formulation, just an algebraic transformation of the one above, is also instructive in the meaning of the delta symbol. In this case, we have the difference between sample and standard expressed as a ratio, in fact a ratio of the 18/16 ratios. This ratio is always nearly one (for example, 1.0001 or 0.9990). From this ratio, the delta value is a sort of shorthand: by subtracting 1.0000, we effectively lop off the “1”, and then by multiplying by 1000 we move the decimal point three places to the right. Thus the cumbersome ratios 1.0010 and 0.9990 become the more convenient delta values 1.0 and -1.0.